

# Mathematics Education in the Digital Age: progress, reality and directions for future research

Prof Dame Celia Hoyles  
UCL Institute of Education  
University College London  
U.K.

谢谢范教授和他所有棒棒的学生，  
谢谢华东师范大学的邀请。

# structure of my talk

## 1. Reflections on my keynote at ICME, Mexico 2008

**Transforming the mathematical practices of learners and teachers through digital technology**

Research in Mathematics Education, Published online: 04 Jul 2018

With response by Paul Drijvers *Freudenthal Institute, Utrecht University*

collaborative research & mutual exchanges

# 2008 ICME talk: enduring premises.....

- centrality of **representation**

Abstractions are *shaped by* and *expressed in* the medium (Noss & Hoyles, Windows on Mathematical Meanings, 1996)

- semiotic mediation

crucial importance of fostering **mathematical**

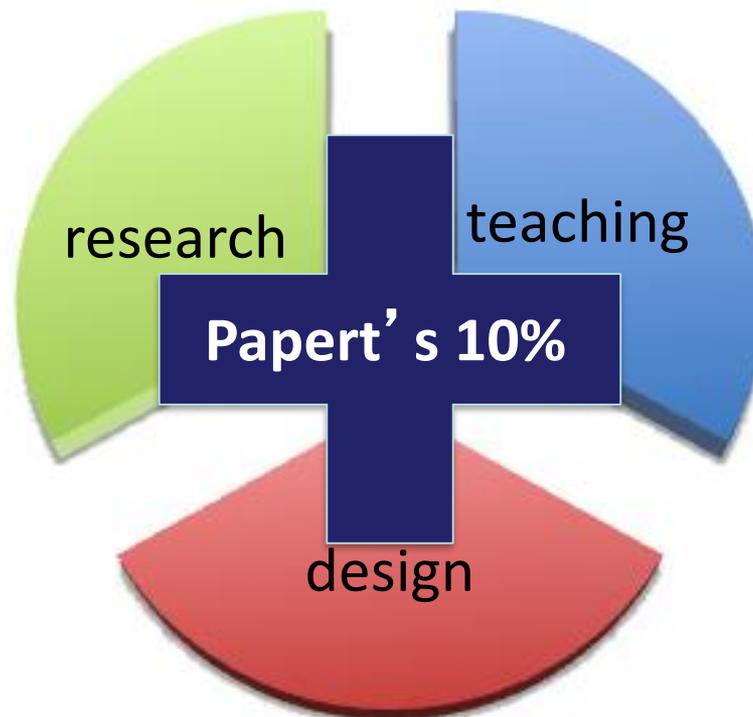
**epistemic agency** ...the group assumes for the ownership of ideas  
(Bereiter, 2002; Scardamalia, 2002)

so

less 'guess what is in the mind of the teacher' (the didactical paradox, Brousseau)

- discuss, share, challenge & reflect
- foster **epistemic agency**

# Exploiting digital technologies for 'better' mathematics learning requires .....



# the potential of digital technology ICME 2008

dynamic & visual **2D & 3D** tools  
to explore in **shared** space

changing how mathematics  
is taught & learned

tools to outsource processing  
power

changing the collective focus of  
attention

new representational  
infrastructures for maths

changing what can be learned and  
by whom

connections between school  
and learners' culture

bridging the gap: school maths and problem  
solving in the 'real world: **joint exploration  
of authentic problems**

connectivity

opening new opportunities for shared  
knowledge construction & student  
autonomy: **genuinely collaborative**

intelligent support for the  
teacher

making **exploratory** learning  
environments more possible

# the potential of digital technology ICME 2008

dynamic & visual **2D & 3D** tools  
to explore in **shared** space

changing how mathematics  
is taught & learned

tools to outsource processing  
power

changing the collective focus of  
attention

new representational  
infrastructures for maths &  
interfaces (**touch, feel ...**)

changing what can be learned and  
by whom

connections between school  
and learners' culture

bridging the gap: school maths and problem  
solving in the 'real world: **joint exploration  
of authentic problems**

connectivity

opening new opportunities for shared  
knowledge construction & student  
autonomy: **genuinely collaborative**

intelligent support for the  
teacher

making **exploratory** learning  
environments more possible

connections between school  
and learners' culture

bridging the gap: school maths and problem  
solving in the 'real world: **joint exploration  
of authentic problems**

Aydin, H. & Monaghan, J. **Encouraging students' problem posing through  
importing visual images into mathematical software** *Teaching Mathematics and  
its Applications: An International Journal of the IMA*, Volume 37, Issue 3, 5  
September 2018

# the potential of digital technology ICME 2008

dynamic & visual **2D & 3D** tools  
to explore in **shared** space

changing how mathematics  
is taught & learned

tools to outsource processing  
power

changing the collective focus of  
attention

new representational  
infrastructures for maths &  
interfaces (**touch, feel ...**)

changing what can be learned and  
by whom

connections between school  
and learners' culture

bridging the gap: school maths and problem  
solving in the 'real world': **joint exploration  
of authentic problems**

connectivity

opening new opportunities for shared  
knowledge construction & student  
autonomy: **genuinely collaborative**

intelligent support for the  
teacher

making **exploratory** learning  
environments more possible

connectivity

opening new opportunities for shared  
knowledge construction & student  
autonomy: **genuinely collaborative**

Huge literature on collaborative learning,  
group work, pair work, peer tutoring

Face-to-face and online

need to design to catalyse sharing of .....

- resources
- information
- student solutions or part-solutions

... generate **mathematical** discussion

- **process** by which knowledge is constructed
- **justifications & refutations**

# the potential of digital technology ICME 2008

dynamic & visual **2D & 3D** tools  
to explore in **shared** space

changing how mathematics  
is taught & learned

tools to outsource processing  
power

changing the collective focus of  
attention

new representational  
infrastructures for maths &  
interfaces (**touch, feel ...**)

changing what can be learned and  
by whom

connections between school  
and learners' culture

bridging the gap: school maths and problem  
solving in the 'real world: **joint exploration  
of authentic problems**

connectivity

opening new opportunities for shared  
knowledge construction & student  
autonomy: **genuinely collaborative**

intelligent support for the  
teacher

making **exploratory** learning  
environments more possible

intelligent support for the  
teacher

making exploratory learning  
environments more possible

- harness **Artificial Intelligence** techniques in the interests of mathematical learning

‘user’ modelling based on mathematics  
education research

learning trajectories more explicit

# the potential of digital technology ICME 2008

dynamic & visual **2D & 3D** tools  
to explore in **shared** space

changing how mathematics  
is taught & learned

tools to outsource processing  
power

changing the collective focus of  
attention

new representational  
infrastructures for maths &  
interfaces (**touch, feel ...**)

changing what can be learned and  
by whom

connections between school  
and learners' culture

bridging the gap: school maths and problem  
solving in the 'real world: **joint exploration  
of authentic problems**

connectivity

opening new opportunities for shared  
knowledge construction & student  
autonomy: **genuinely collaborative**

intelligent support for the  
teacher

making **exploratory** learning  
environments more possible

tools to outsource processing  
power

changing the collective focus of  
attention

• Example

easy and free graph-drawing software means students can use quick (and internet browser friendly) **graph plotting** for checking and challenging incorrect thinking

**does losing the skill of *graph sketching* matter?**

# the potential of digital technology ICME 2008

dynamic & visual **2D & 3D** tools  
to explore in **shared** space

changing how mathematics  
is taught & learned

tools to outsource processing  
power

changing the collective focus of  
attention

new representational  
infrastructures for maths &  
interfaces (**touch, feel ...**)

changing what can be learned and  
by whom

connections between school  
and learners' culture

bridging the gap: school maths and problem  
solving in the 'real world': **joint exploration  
of authentic problems**

connectivity

opening new opportunities for shared  
knowledge construction & student  
autonomy: **genuinely collaborative**

intelligent support for the  
teacher

making **exploratory** learning  
environments more possible

new representational  
infrastructures for maths

changing what can be learned and  
by whom



...the difficulty of a mathematical  
idea often derives from the  
system with which it is expressed

—**imagine just how difficult it  
would be to remember the  
various procedural rules of  
calculus (like the chain rule)  
without Leibniz's elegant  
notation.** (Kaput, Hoyles & Noss,  
2002)

# Coding: an international phenomenon

Eric Schmidt  
Chief Executive of Google  
visited England 2011



*“I was flabbergasted to learn that today Computer Science isn't even taught as standard in UK schools”*

*“Your IT curriculum focuses on teaching how to use software, **but gives no insight into how it's made**”.*

Everybody  
needs to  
program

theguardian | theobserver

News | Sport | Comment | Culture | Business | Money | Life & style | Travel

News > Education > Computer science and IT

Why all our kids should be taught how

# Bring coding into **school maths** ScratchMaths project 😊

New Computing  
National  
Curriculum in  
England  
2014



John Naughton

The Observer, Saturday 31 March 2012 20.15 BST

 Jump to comments (288)



## The ScratchMaths project

**design research** to develop a 2-year curriculum for 9-11 year olds in England

- aligned to the national computing *and* national

Computational thinking

*and*

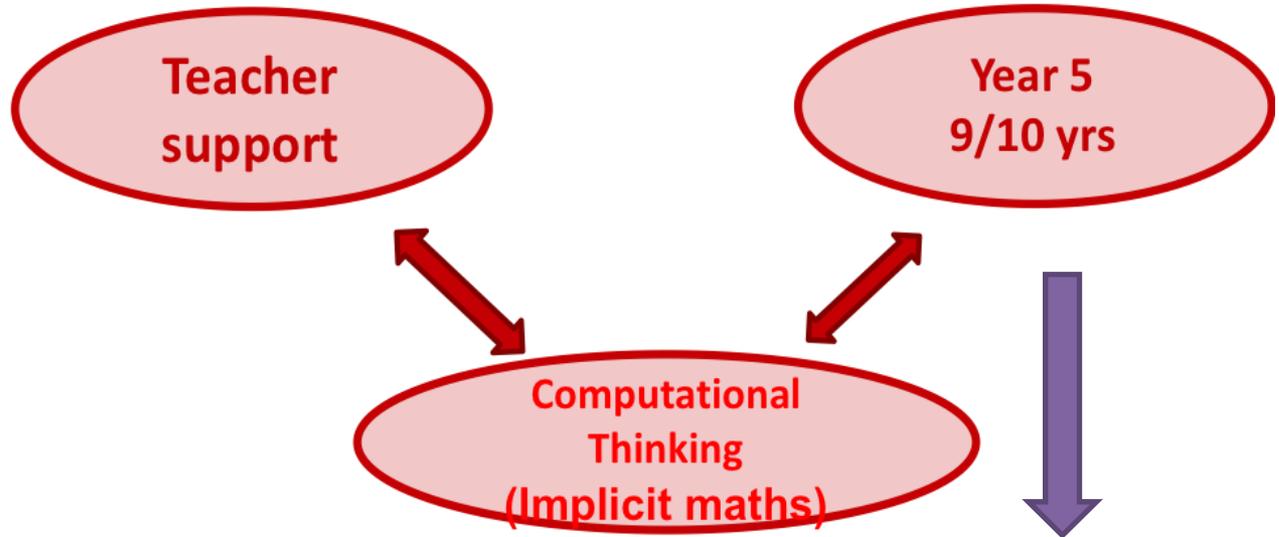
Mathematical thinking

**Method:** iterative design and trialling in four ‘design schools’

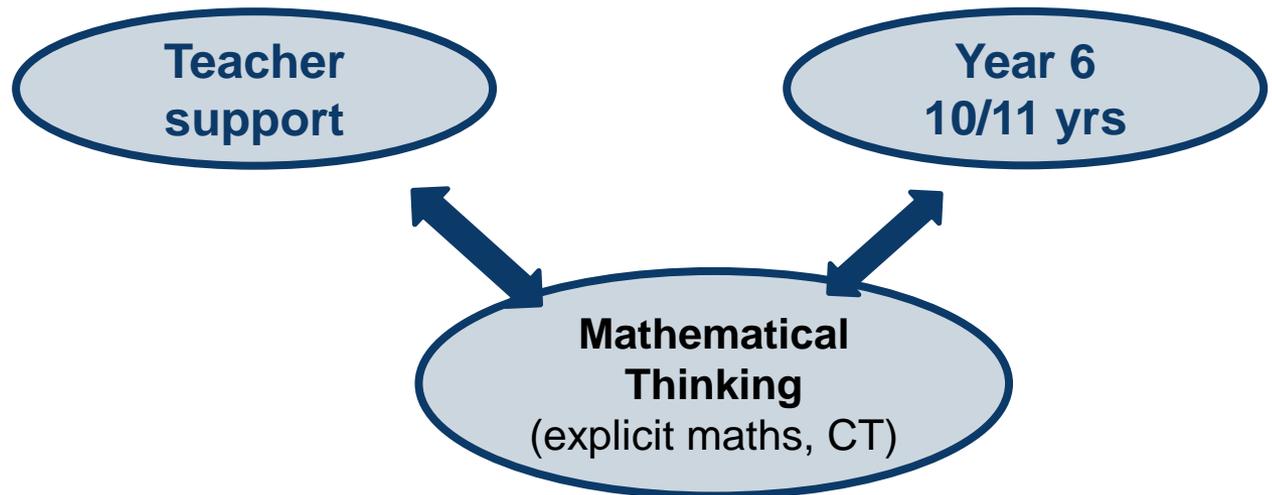
# **Glimpse of the classroom research case studies**

# Two-year “instructional sequence”

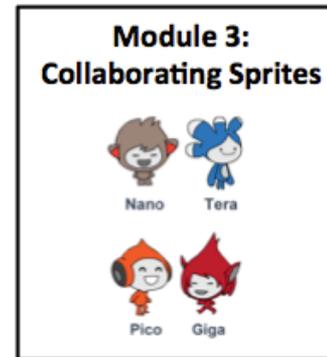
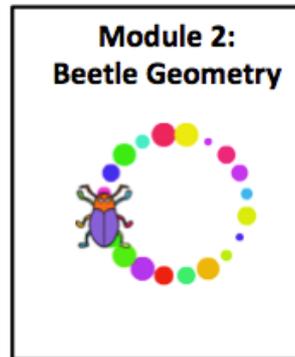
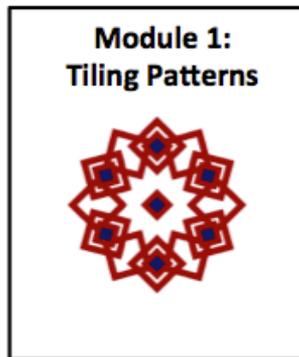
**Intervention  
Year1**



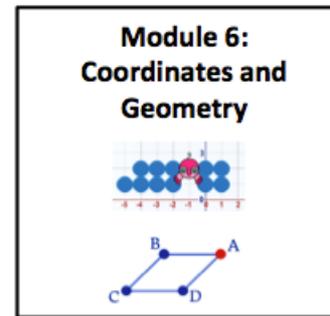
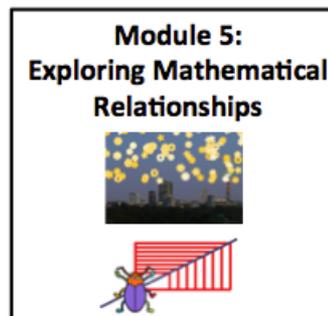
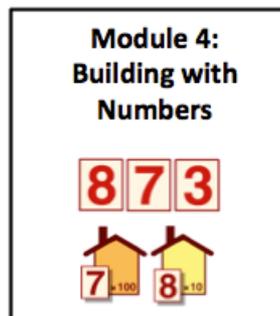
**Intervention  
Year2**



## Year 5 (9-10 yrs) – Computing focus (20+ hours of teaching materials)



## Year 6 (10-11 yrs) – Mathematics focus (20+ hours of teaching materials)



# Extensive Teacher Support Materials and Example Scripts

## MODULE 1: Tiling Patterns

**Investigation 1**  
Moving, Turning and Stamping

**Investigation 2**  
Repeating and Alternating Patterns

**Investigation 3**  
Creating Circular Rose Patterns

**Investigation 4**  
Defining Your Own Pattern Blocks

### INTRODUCTION TO MODULE 1

The theme of Module 1 is **repeating patterns**. You may like to introduce this module by linking it to another area of the curriculum such as art or science where similar patterns can be found. Some examples are below.

**ART: ISLAMIC OR GOTHIC ART**  
Geometric patterns have been used extensively in Islamic art for many centuries and can also be found in gothic architectural features, such as stained glass windows.

**SCIENCE: PATTERNS IN NATURE**  
Geometric patterns are seen in nature for example in snowflakes or in the sand sculptures created by puffer fish on the ocean floor.

#### KEY VOCABULARY AND CONCEPTS COVERED BY MODULE 1

Block	Command	Mathematics
<ul style="list-style-type: none"> <li>Sprite</li> <li>Stage</li> <li>Block</li> <li>Stamp block</li> <li>Hot block</li> <li>Turn block</li> <li>Scoping blocks</li> <li>Script</li> <li>Move block</li> <li>Repeat block</li> <li>Define block</li> </ul>	<ul style="list-style-type: none"> <li>Command</li> <li>Program, programming</li> <li>Debugging</li> <li>Sequence</li> <li>Repetition</li> <li>Logic of reasoning</li> <li>Algorithm</li> <li>Definition</li> </ul>	<ul style="list-style-type: none"> <li>Symmetry</li> <li>Translation</li> <li>Angles (right, obtuse, reflex and acute)</li> <li>Patterns</li> <li>Rotation</li> <li>Transformation</li> <li>Sequences</li> <li>Positive and negative numbers</li> <li>Coordinates</li> </ul>

## MODULE 1: INVESTIGATION 1 Moving, Turning and Stamping

**OVERALL LEARNING OBJECTIVE:** Drag, turn, move and stamp a sprite, and build a simple script to create a pattern without using unnecessary blocks.

This investigation introduces three important Scratch commands (**move**, **turn** and **stamp**) and gradually builds to using these in a program (**script**) to create a simple pattern. The investigation comprises of four activities.

- Activity 1.1.1 - Drag and Stamp
- Activity 1.1.2 - Drag, Turn and Stamp
- Activity 1.1.3 - Move, Turn and Stamp
- Activity 1.1.4 - Unplugged Simple Sorters

Activity 1.1.1: 10-15 mins | Activity 1.1.2: 15-20 mins | Activity 1.1.3: 10-15 mins | Activity 1.1.4: 10-15 mins

We recommend allowing 60 to 90 minutes for this investigation.

**Scratch starter projects:**

- 1-The Stamp
- 1-The Turn
- 1-The Move

#### LINKS TO PRIMARY NATIONAL CURRICULUM

Curriculum Description	Link with Scratch/Make
<p><b>Computing</b></p> <p>Design, write and debug simple programs that accomplish specific goals.</p>	<ul style="list-style-type: none"> <li>Pupils are required to create a script that produces a pattern.</li> </ul>
<p><b>Mathematics</b></p> <p>Identify lines of symmetry in 2D shapes presented in different orientations.</p> <p>Identify, describe and represent the position of a shape following a translation.</p> <p>Recognise angles as a description of a turn, inner angles are measured in degrees, identify different kinds of angles and use right angle facts.</p>	<ul style="list-style-type: none"> <li>Pupils are required to create patterns with one or more lines of symmetry.</li> <li>Pupils are required to move and stamp their sprite to create patterns.</li> <li>Pupils are required to use their knowledge of angles to rotate their sprite and create different patterns.</li> </ul>

## MODULE 1 • INVESTIGATION 1 • ACTIVITY 1.1.1 Drag and Stamp

**Explore** how to drag and stamp a sprite to create symmetrical patterns. Explain what is happening when the green flag is clicked.

**Activity Instructions:**

- Pupils open project 1-The Stamp, save as a copy and add their name(s) to the title.
- Pupils create a symmetrical pattern by dragging the Tilted sprite and clicking on the stamp block in the Scripts area.
- Pupils can save their pattern by right-clicking (or Shift + click) on the stage and selecting save picture of stage.
- Pupils click on the green flag to run the setup script - this resets the stage and the sprite.

**Checklist for Pupils:**

- The setup script should not be modified.
- Stamped patterns are not saved in your project - you can only save a picture of the whole stage.
- You need to click on the stamp block carefully to make sure it runs (look to see if the green flag flashes).

**Checklist for Teachers:**

- How many stamps have you used?
- What colour is the stamp block? Which group of blocks does it belong to? Where can we find it?
- Did you have any problems with stamping?
- Have you clicked on the green flag to start again? What happens? Why do you think it does this? (go to centre of stage)
- When the sprite is moved what happens to the x,y coordinates?
- What makes your pattern symmetrical?
- How many lines of symmetry does your pattern have?

**Discussion:**

A **sprite** is an object. The **stage** is where you can see the sprites. A **block** represents a command which tells the sprite what to do and you can run by clicking on it. The **stamp** block tells the sprite to print an image of itself on the stage. The **whom green flag** (clicked block) is called a **hot block**, it is always placed at the top of a script.

## INVESTIGATION 1 Activity 1.1.1

This screenshot shows the names given to the different areas of the Scratch interface.



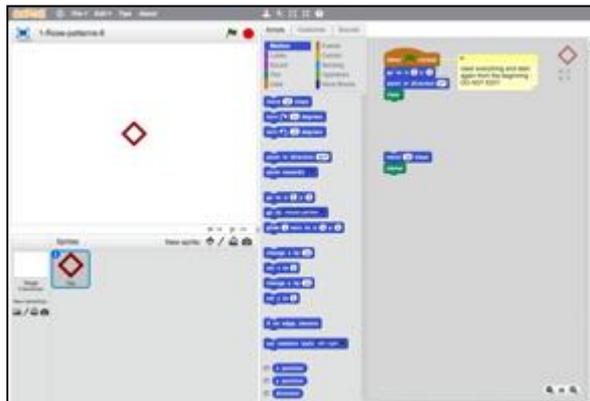
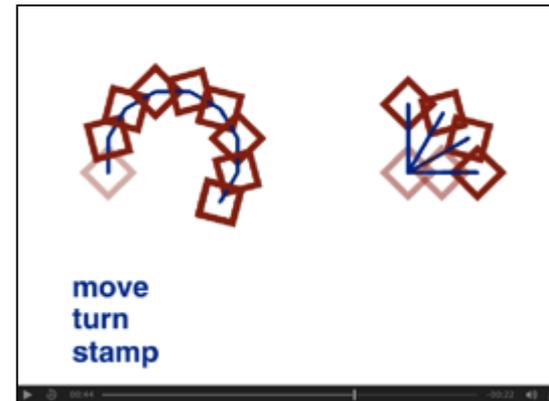
Use this cut-out to show your pupils what the Tilted sprite is and that is pointing north.

In the later activities it can be used to demonstrate turning and moving.



When using the cut-out to demonstrate turning it is important to show the relation point at the bottom of the tile.

## Scratch Starter Projects

move  
turn  
stamp

## Support Videos

## Presentation Slides for every lesson

**BETLE GEOMETRY**  
**MODULE 2: INVESTIGATION 3**

### Discovering Dots

**MODULE 2: INVESTIGATION 3**  
**Activity 2.3.2 – Unplugged: Picture Predictions**

Read each of the scripts. Draw and/or explain in words the picture that it will create.

**MODULE 2: INVESTIGATION 3**  
**Activity 2.3.3 – Swarming Dots**

Replace the **move** and **turn** blocks in your script with the **jump to random position** block from the **More Blocks** group and run the script.

Try switching the backdrop to **night** or **day** by using the **switch backdrop to ...** block.

## Additional challenges, vocabulary and reference posters

**MODULE 2 • CHALLENGE 4**  
**AROUND THE GLOBE CHALLENGE**

In Module 2 we used the "move forward" + stamp + move backward + turn" algorithm to create circular patterns with the sprite starting and finishing in the center. The same algorithm may help us now to create different circular scenes.

Combine drawing a larger dot with drawing houses, trees, mushrooms or other objects around the outside edge. Be sure that the Beetle always returns back to the same position in the center, then turns a bit and repeats.

**MODULE 2 • VOCABULARY**

- pen tool** each sprite has a pen tool and can draw lines on the stage when its pen tool is down.
- set pen blocks** allow you to change the colour, size (width) and shade of the line that is drawn.
  - set pen colour to
  - set pen size to
  - set pen shade to
- pen down** after running this block, the sprite will continuously draw a trail wherever it moves (until pen up block is used).
- pen up** after running this block, the sprite will stop drawing a trail while it moves (until pen down block is used again).
- backdrop** the background of the stage: there can be multiple backdrops and the stage can change its look to display any of them by
  - switch backdrop to color
- pick random** picks a random integer number within a specified interval. For example can be used in
  - set pen size to pick random to 50

**MODULE 2 • POSTER 2**  
**SCRATCH COLOUR SCHEME – PEN SHADES**

Each colour that you set using **set pen colour to** or **pick random color** may have 100 different pen shades. If the pen shade is 0, then the colour will be very dark. If the pen shade is 100, the colour will be close to white. The default pen shade setting (that is, when you do not set it at all) is 50, a shade in the middle between very dark and very light.

pen shade

# Pedagogical framework (5Es)

**Explore:** Investigate, try things out yourself, debug in reaction to feedback

**Envisage:** Have a goal in mind, predict outcome of program *before trying it*

**Explain:** Explain what you have done, articulate reasons behind your approach to yourself & others

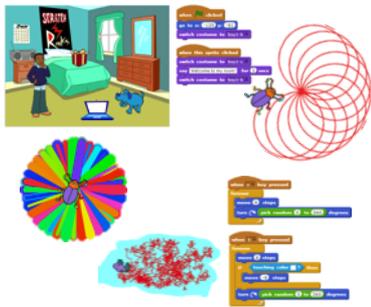
**Exchange:** Collaborate & share, try to see a problem from another's perspective as well as defend your own approach and compare with others.

**bridgE:** Make **explicit** links to the mathematics curriculum

# **Glimpse of the classroom research case studies**

**Quantitative results in Jan 2019**

# Overview of SM research process



- Theory and literature
- Design workshops
- School observations

- Observed intervention 4 schools
- Discussions with pupils and teachers

- Two days of professional development
- Discussions with teachers

- Pre/post surveys
- Observed same activity in a subset of schools
- Interviewed teachers and pupils

# The ScratchMaths project...

## National trial

- 110 English primary schools recruited in early 2015, 2,986 students

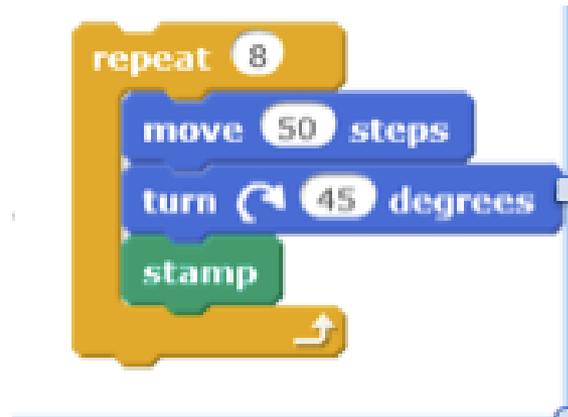


Control schools put on 'wait list' to receive intervention one year after treatment schools

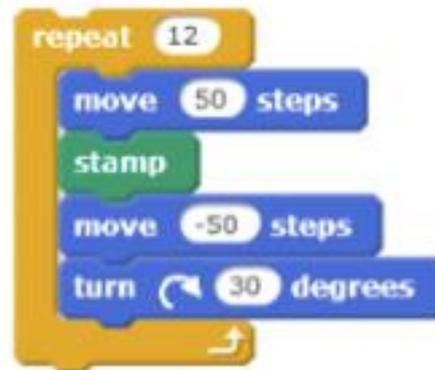
prior attainment as measured by national standardised mathematics assessment at age 8 years

# example: Compare two algorithms

Algorithm 1



Algorithm 2



two learning goals

- notion of algorithm
- 360 as total turn



```
repeat 8  
  move 50 steps  
  turn 45 degrees  
  stamp
```

```
repeat 12  
  move 50 steps  
  stamp  
  move -50 steps  
  turn 30 degrees
```

# Unplugged tasks....

Which algorithm was used to create each of the drawings?

## Algorithm 1

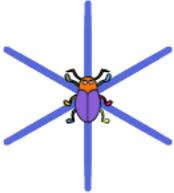
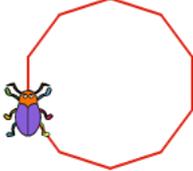
Repeat the following:

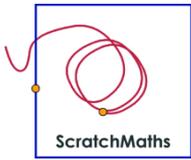
- Move
- Turn

## Algorithm 2

Repeat the following:

- Move
- Move backwards
- Turn

Drawing	Which algorithm does it use?
1 	
2 	
3 	
4 	



## Module 2: Investigation 3

### Activity 2.3.2 – Unplugged: Picture Predictions



- Read each of the scripts. Draw and/or explain in words the picture that it will create.

1

```
clear
set pen color to red
set pen size to 10
repeat 24
  dot
  move 20 steps
  turn 15 degrees
```

2

```
clear
set pen color to red
set random pen size
repeat 24
  dot
  move 20 steps
  turn 15 degrees
```

3

```
clear
set pen color to red
repeat 24
  set random pen size
  dot
  move 20 steps
  turn 15 degrees
```

4

```
clear
set pen color to red
repeat 24
  set random pen size
  set random pen colour
  dot
  move 20 steps
  turn 15 degrees
```

?

?

?

?



Read each of the scripts. Draw and/or explain in words the picture that it will create.

1

```
clear
set pen color to red
set pen size to 10
repeat 24
  dot
  move 20 steps
  turn 15 degrees
```

2

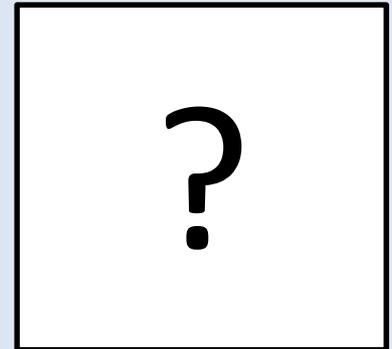
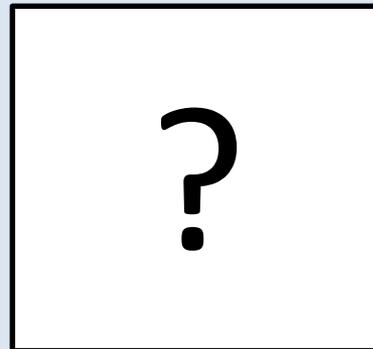
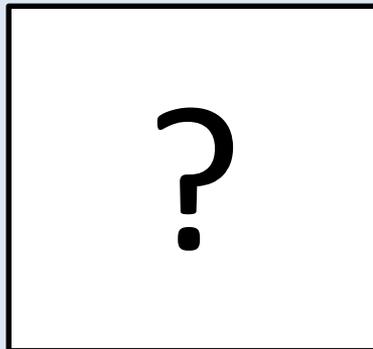
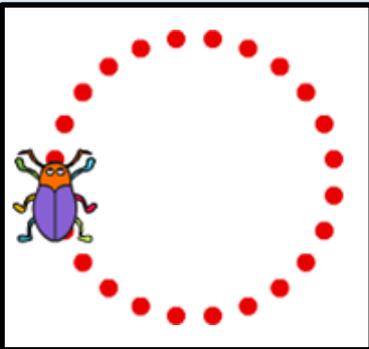
```
clear
set pen color to red
set random pen size
repeat 24
  dot
  move 20 steps
  turn 15 degrees
```

3

```
clear
set pen color to red
repeat 24
  set random pen size
  dot
  move 20 steps
  turn 15 degrees
```

4

```
clear
set pen color to red
repeat 24
  set random pen size
  set random pen colour
  dot
  move 20 steps
  turn 15 degrees
```





Read each of the scripts. Draw and/or explain in words the picture that it will create.

1

```

clear
set pen color to red
set pen size to 10
repeat 24
  dot
  move 20 steps
  turn 15 degrees
  
```

2

```

clear
set pen color to red
set random pen size
repeat 24
  dot
  move 20 steps
  turn 15 degrees
  
```

3

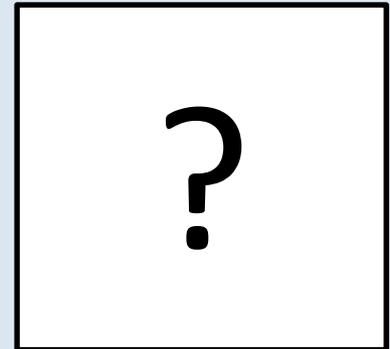
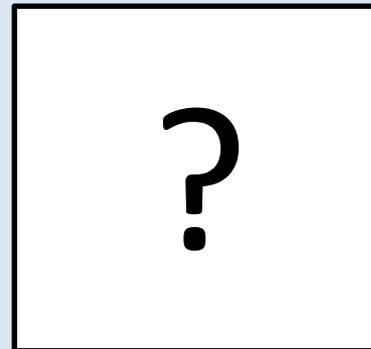
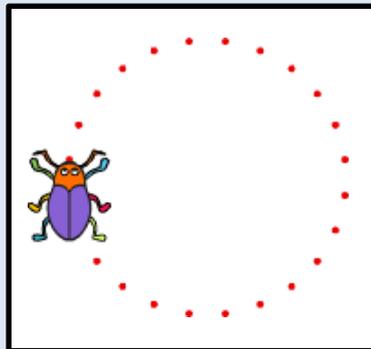
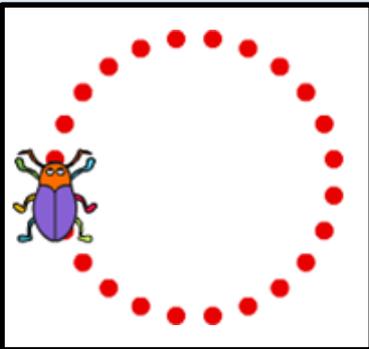
```

clear
set pen color to red
repeat 24
  set random pen size
  dot
  move 20 steps
  turn 15 degrees
  
```

4

```

clear
set pen color to red
repeat 24
  set random pen size
  set random pen colour
  dot
  move 20 steps
  turn 15 degrees
  
```



## Module 2: Investigation 3

### Activity 2.3.2 – Unplugged: Picture Predictions



Read each of the scripts. Draw and/or explain in words the picture that it will create.

1

```
clear
set pen color to red
set pen size to 10
repeat 24
  dot
  move 20 steps
  turn 15 degrees
```

2

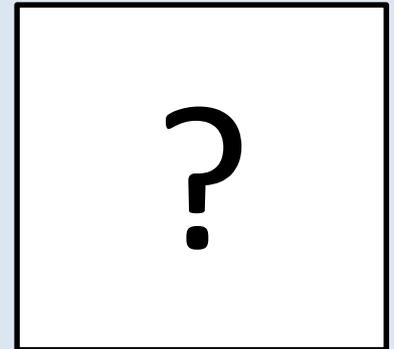
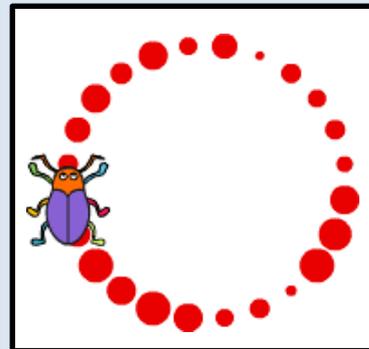
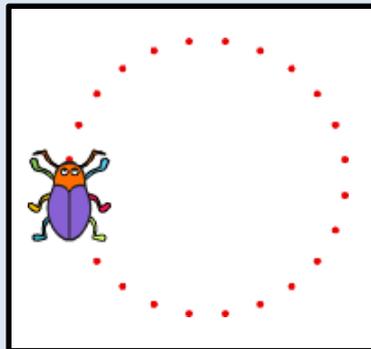
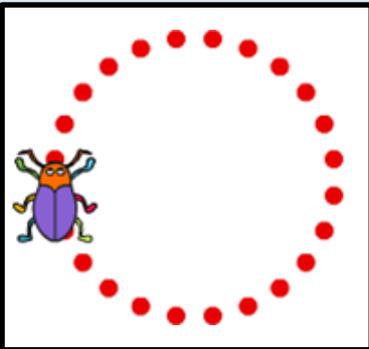
```
clear
set pen color to red
set random pen size
repeat 24
  dot
  move 20 steps
  turn 15 degrees
```

3

```
clear
set pen color to red
repeat 24
  set random pen size
  dot
  move 20 steps
  turn 15 degrees
```

4

```
clear
set pen color to red
repeat 24
  set random pen size
  set random pen colour
  dot
  move 20 steps
  turn 15 degrees
```



## Module 2: Investigation 3

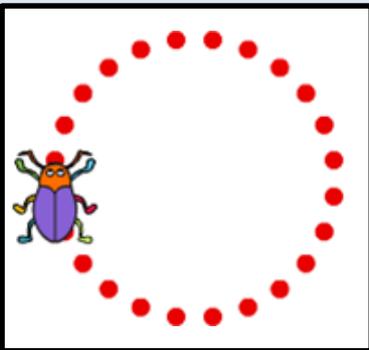
### Activity 2.3.2 – Unplugged: Picture Predictions



Read each of the scripts. Draw and/or explain in words the picture that it will create.

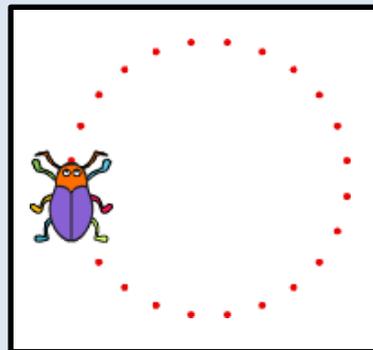
1

```
clear
set pen color to red
set pen size to 10
repeat 24
  dot
  move 20 steps
  turn 15 degrees
```



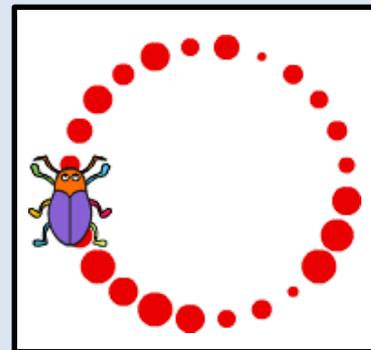
2

```
clear
set pen color to red
set random pen size
repeat 24
  dot
  move 20 steps
  turn 15 degrees
```



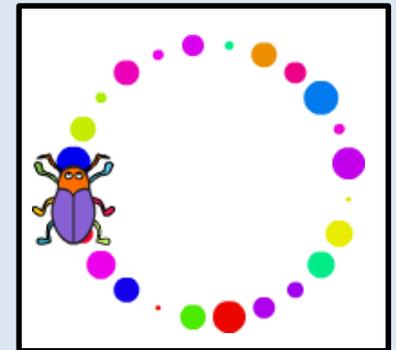
3

```
clear
set pen color to red
repeat 24
  set random pen size
  dot
  move 20 steps
  turn 15 degrees
```



4

```
clear
set pen color to red
repeat 24
  set random pen size
  set random pen colour
  dot
  move 20 steps
  turn 15 degrees
```



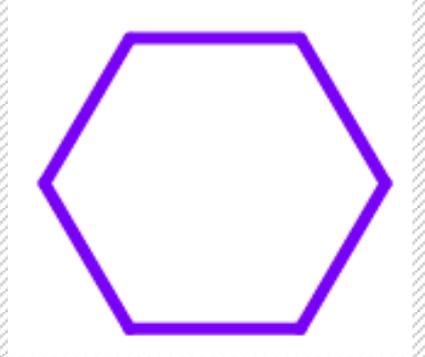
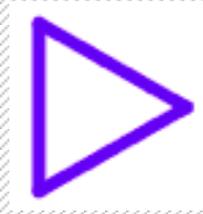
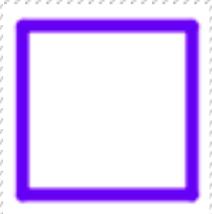
Match the script to the polygon it would draw.

```
set pen size to 5
pen down
set pen color to purple
repeat 4
  move 70 steps
  turn 90 degrees
```

```
set pen size to 5
pen down
set pen color to purple
repeat 3
  move 70 steps
  turn 120 degrees
```

```
set pen size to 5
pen down
set pen color to purple
repeat 6
  move 70 steps
  turn 60 degrees
```

```
set pen size to 5
pen down
set pen color to purple
repeat 2
  move 120 steps
  turn 90 degrees
  move 70 steps
  turn 90 degrees
```



Explain how you worked out your answer...

## Polygon Fireworks



# fruitful areas of SM research

- Understanding of algorithm (pupil and teacher)
- Affordances of computer programming for mathematics learning
- Evolution of teacher knowledge through engaging with SM
- Pupil and teacher creativity
- ... **and many more**
  - engagement of girls
  - nature of teacher **professional development**
  - assessment & evaluation
  - teacher knowledge & confidence
  - teacher beliefs
  - mathematics anxiety of teachers and students...

• The gap between curriculum design & implementation:  
idea of **fidelity**

**Fidelity:** how far is an innovation implemented according to its aims and objectives?

SM derived 5 criteria as proxy measures of fidelity

1. technology access
2. curriculum progression
3. curriculum coverage
4. teaching time allocated
5. engagement in professional development (PD)

Measures 3,4,5 differentiated *high, medium* and *low* fidelity

# Some findings...from survey data

- fidelity very high in first year but dropped dramatically in second year..

## Why?

- negative impact of the high-stakes mathematics testing in mathematics at the end of school year
- teachers more able to adapt their teaching to E's in the context of teaching a 'new' subject, **computing**, while struggled to change their practice in established and higher-stakes subject, mathematics
- and they lacked confidence in maths & computing

Scratchmaths in other countries

# Some findings...from survey data

Lethal mutations ☹️

**Quantitative results of randomized control trial in Jan 2019**

while struggled to change their practice in established and higher-stakes subject, mathematics, in which many also lacked confidence

# Some findings...from survey data

- fidelity very high in first year but dropped dramatically in second ...

## Why?

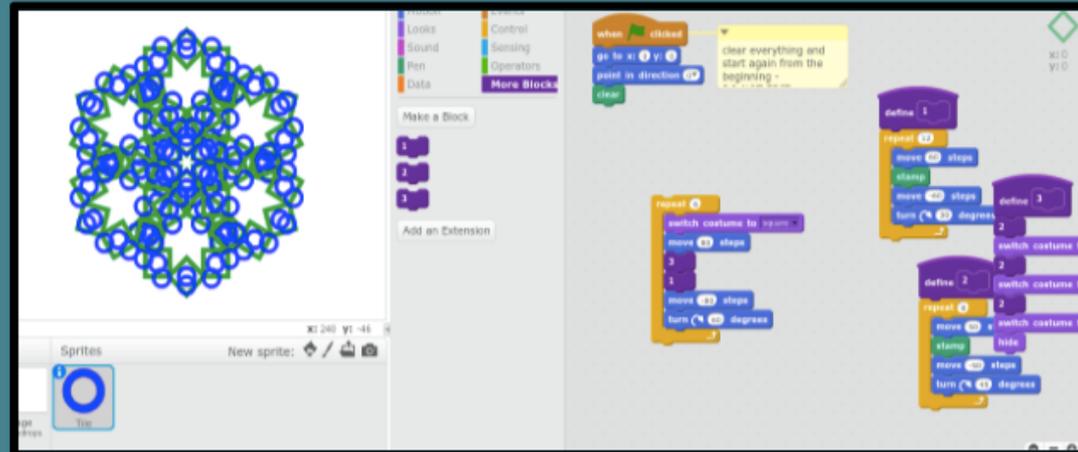
- negative impact of high-stakes testing in mathematics at the end of school year
- teachers felt more able to adapt their teaching to E's in the context of teaching a 'new' subject, **computing**, while struggled to change their practice in established and high-stakes subject, mathematics, ...in which many also lacked confidence

Scratchmaths in other countries

# Scratchmaths in Australia

## Kid FG

Absolutely loved ScratchMaths, it was so fun learning all about coding and was something I looked forward to every Monday. I enjoy having to find the problem when it goes wrong and then making the code much better. I find it really cool that you can program a computer to do something like turning a certain amount of degrees or moving a certain amount of steps. ScratchMaths was difficult at first but once I learnt the basics all I wanted to do is learn



Led by Elena Prieto-Rodriguez & Kathryn Holmes  
And in USA led by Paul Goldenberg  
And many European countries – and ....



# Scratchmaths in China ctd

Dr. Hongliang Ma, Professor in Educational Technology,  
Shaanxi Normal University

- designed ScratchMaths learning materials aligned to the Chinese Math Curriculum Standard for grade four students
- is supervising experimental research in a primary school based on the 5Es' pedagogy of ScratchMaths.

What are differences & similarities in implementation & outcomes?

# the potential of digital technology ICMF 2008

dynamic & visual **2D & 3D** tools  
to explore in **shared** space

changing how mathematics  
is taught & learned

tools to outsource processing  
power

changing the collective focus of  
attention

new representational  
infrastructures for maths &  
interfaces (**touch, feel ...**)

changing what can be learned and  
by whom

connections between school  
and learners' culture

bridging the gap: school maths and problem  
solving in the 'real world: **joint exploration  
of authentic problems**

connectivity

opening new opportunities for shared  
knowledge construction & student  
autonomy: **genuinely collaborative**

intelligent support for the  
teacher

making **exploratory** learning  
environments more possible

dynamic & visual **2D & 3D** tools  
to explore in **shared** space

changing how mathematics  
is taught & learned

# Shift from tasks to research-informed curriculum innovation

Example: Cornerstone Maths

# Cornerstone mathematics

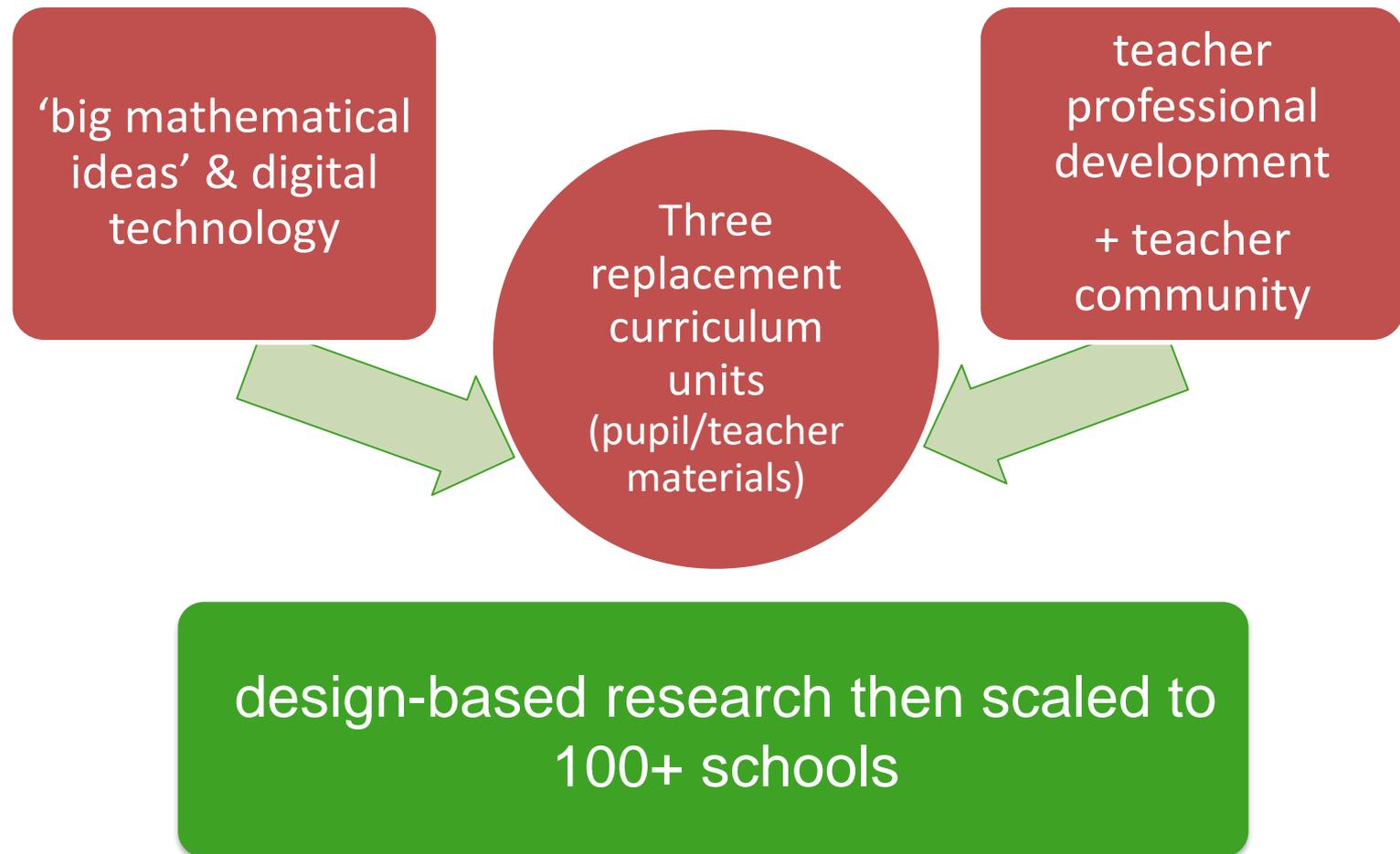
Celia Hoyles, Richard Noss, Alison Clark-Wilson & SRI International (Roschelle and Vahey)



exploits the dynamic and visual nature of digital technology for students aged 11-14 years to stimulate engagement with mathematical ways of thinking by

- focusing on ‘big mathematical ideas’ that are hard to teach
- making dynamic links between key representations
- providing an environment for students to explore & solve problems within guided structured activities

# Cornerstone Maths: a curriculum innovation approach

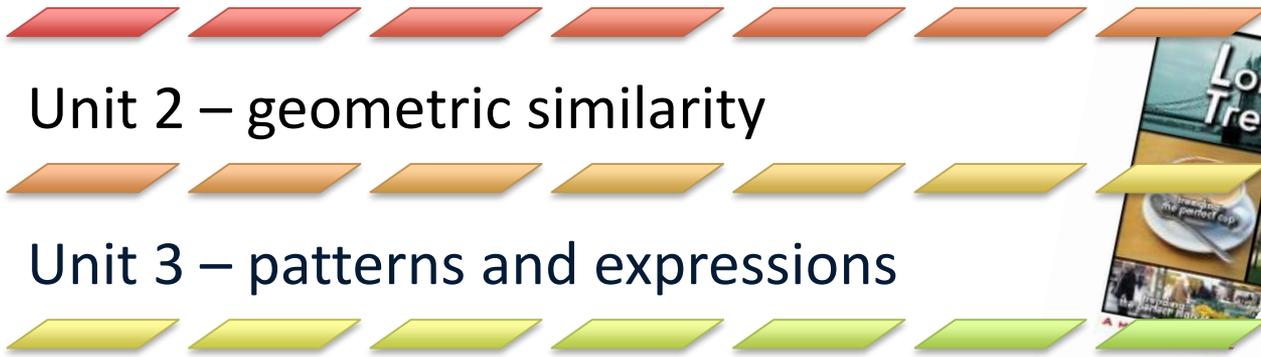


# The Cornerstone Maths units

Unit 1 – linear functions

Unit 2 – geometric similarity

Unit 3 – patterns and expressions



CORNERSTONE MATHS

CORNERSTONE MATHS

## Lethal mutations

Brown, A. L., & Campione, J. C. (1996). *Psychological theory and the design of innovative learning environments: On procedures, principles, and systems*: Lawrence Erlbaum Associates, Inc.

## Landmark activity

- Use of digital technology leads to
  - cognitive breakdown
  - 'situation of non-obviousness'
  - 'aha' moment: surprise, rethinking
- suggests growing appreciation of underlying concept(s)

**maybe not as anticipated**



Context: a company creating video games for mobile phones

*"You are helping this company to design new games"*

# Designing Mobile Games

A module on linear functions



Jim Kaput,  
SimCalc

Jeremy  
Roschelle &  
SRI  
collaboration

# Landmark

**A glimpse of Cornerstone in action**

# Potential research areas....

## Instrumentation (fluency with digital tools)

Prof SC Kong Director of the Center for Learning, Teaching and Technology and Prof of Math and IT Hong Kong Education University

Organised a Summer camp around using Cornerstone

What are factors fostering 'spread'

Cornerstone is used within USA and many European countries and.....  
China

Potential for cross-country research?

# More cycles of design research using 'new' tools

- Augmented reality
- Multi-touch
- Mobile devices with touch screens,
- 3D visualisations
- ....

Ref: Recent research on geometry education: ICME-13 survey team report Nathalie Sinclair, Maria G. Bartolini Bussi, Michael de Villiers, Keith Jones, Ulrich Kortenkamp, Allen Leung, Kay Owens

dynamic & visual **2D & 3D** tools  
to explore in **shared** space

changing how mathematics  
is taught & learned

# A Confession about Personalized Learning

Larry Berger, CEO of Amplify

With thanks to Paulo Blickstein

# In summary..

- Digital tools are creeping into schools at varying rates...
  - which classes? age and ‘ability’? presentation? flipped classrooms etc. Is tool-use transformational?

Hoyles, C., (1993) *Microworlds/Schoolworlds: The transformation of an innovation*. In Keitel, C., Ruthven, K. (eds) *Learning from Computers: Mathematics Education and Technology*. NATO ASI, Series F: Computer and Systems Sciences, 121, 1-17.

- Semiotic mediation
- Networking theories
- Embodiment; gesture; tool use as perceptuo-motor fluency

**What new insights do the use of such theories bring?**

## Challenges to SM in England....

- complexity of the process of integrating digital technology (solid findings in Mathematics Education: European Mathematics Society newsletter March 2014)
- shortage of mathematics teachers, 'teacher churn'

**“If you’re the head of maths you are so accountable: if the maths department goes down, the whole school goes down.”**

## Challenges to SM in England ctd...

- curriculum change
- high stakes mathematics testing
- widespread mathematics anxiety
- Shanghai mathematics initiative

# Shanghai initiative

**Is Shanghai initiative compatible with curriculum initiatives that embed digital technologies such as ScratchMaths or Cornerstone Maths?**

## Shanghai visit for minister to learn maths lessons

By Patrick Howse  
BBC News, Education reporter

## Britain's schools need a Chinese lesson

A visit to Shanghai's classrooms confounds our every expectation about Asian maths teaching

Women Fa  
land | Roya  
ables | Profes



Thank you

谢谢

Questions or comments please